COMBINED HEAT AND MASS TRANSFER DURING ABSORPTION IN DROPS AND FILMS

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Combined heat and mass transfer during absorption in films and drops is discussed. Using an approximate approach, simple analytic relations are obtained for the basic characteristics of the process.

The physical processes which take place during absorption by liquid drops or films moving in a medium containing an absorbable material are extremely diverse and complex. In studying them it is therefore rather useful to have rather simple and predictable models which give a qualitatively true reflection of them although failing to reproduce the detailed pattern of the phenomena.

We consider the problem of combined heat and mass transfer during absorption of vapor by drops of fluid under the following assumptions. The atmosphere in which the drop is located contains no noncondensable gases. The drop is stationary with respect to the surrounding vapor atmosphere and is spherical in shape. At the initial time t = 0, a state of saturation is established instantaneously over the entire surface of the drop for the absorbed-material—liquid-solution system which is subsequently maintained throughout the entire process.

The dependence of concentration C on temperature T in the state of saturation is linear, C = dT + b.

All physical parameters of the problem (coefficients of thermal conductivity, diffusion, etc.) are constant over the ranges of temperatures and concentrations considered.

Under the assumptions made, heat and mass transfer is described by the following system of equations:

$$\begin{aligned} \frac{\partial T}{\partial t} &= a\left(\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}\right), \\ \frac{\partial C}{\partial t} &= D\left(\frac{\partial^2 C}{\partial r^2}+\frac{2}{r}\frac{\partial C}{\partial r}\right) \end{aligned} \tag{1}$$

with boundary conditions of the form

$$C(t,\ R)=dT(t,\ R)+b, \tag{2}$$

$$\lambda\left.\frac{\partial T}{\partial r}\right|_{r=R}=\rho r_a D\left.\frac{\partial C}{\partial r}\right|_{r=R}. \tag{3}$$

We take

$$T(0,\ r)=T_0,\quad C(0,\ r)=C_0$$

as initial conditions for Eqs. (1), where C_0 is less than the saturation value corresponding to the temperature T_0.

We also assume that the process of heat and mass transfer is localized near the surface of the drop within the limits of a layer with a thickness less than the radius of the drop.

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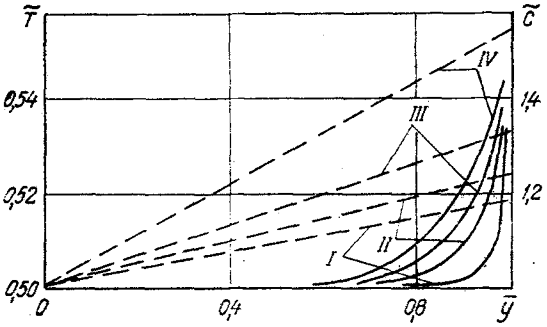

Fig. 1. Profiles of temperature (dashed lines) and concentration (solid lines) at various cross sections of a film (Le = 25; T_w/T_0 = 0.5; dT_0/C_0 = −0.5; b/C_0 = 1.7; Ka = −0.1): I) $\bar{x}$ = 0.03; II) $\bar{x}$ = 0.09; III) $\bar{x}$ = 0.21; IV) $\bar{x}$ = 0.3.

The possibility of a surface-layer approximation then follows. In that case, the initial conditions transform into boundary conditions at the boundary of an asymptotic surface layer, i.e., one can write

$$T|_{r\to-\infty} = T_0; \tag{4}$$

$$C|_{r\to-\infty} = C_0. \tag{5}$$

For the surface-layer approximation we also assume

$$C(t,\ R) = C_s(t); \tag{6}$$

$$T(t,\ R) = T_s(t). \tag{7}$$

The temperature and concentration at the surface of the drop (T_s, C_s) are determined during the solution of the problem.

By means of the substitutions $\bar{C} = (C - C_0)r$ and $\bar{T} = (T - T_0)r$, the system (1) with the boundary conditions (4)-(7) reduces to a problem allowing a self-similar solution of the form

$$\bar{C} = \frac{2R(C_s - C_0)}{\sqrt{\pi}} \int\limits_0^{\eta_1} \exp(-x^2)\,dx + R(C_s - C_0),$$

$$\bar{T} = \frac{2R(T_s - T_0)}{\sqrt{\pi}} \int\limits_0^{\eta_2} \exp(-x^2)\,dx + R(T_s - T_0).$$

Here C_s and T_s are considered constant during the integration. In the dimensionless variables

$$\tilde{C} = \frac{C}{C_0};\quad \tilde{T} = \frac{T}{T_0};\quad \mathrm{Fo} = \frac{a}{R^2}\,t;\quad x = \frac{r}{R}$$

this solution takes the form

$$\tilde{C} = \frac{\tilde{C}_s - 1}{x}\ \mathrm{erf}\left[\frac{\sqrt{\mathrm{Le}}\,(x-1)}{2\sqrt{\mathrm{Fo}}}\right] + \frac{\tilde{C}_s - 1}{x} + 1; \tag{8}$$

$$\tilde{T} = \frac{\tilde{T}_s - 1}{x}\ \mathrm{erf}\left[\frac{x-1}{2\sqrt{\mathrm{Fo}}}\right] + \frac{\tilde{T}_s - 1}{x} + 1. \tag{9}$$

We obtain the following expressions for $\tilde{T}_s$ and $\tilde{C}_s$ from conditions (2) and (3):

$$\tilde{T}_s = \frac{\dfrac{C_0 - b}{dT_0}\left(\dfrac{\sqrt{\mathrm{Le}}}{\sqrt{\pi\,\mathrm{Fo}}} - 1\right) - \dfrac{C_p\mathrm{Le}}{r_a d}\left(\dfrac{1}{\sqrt{\pi\,\mathrm{Fo}}} - 1\right)}{\left(\dfrac{\sqrt{\mathrm{Le}}}{\sqrt{\pi\,\mathrm{Fo}}} - 1\right) - \dfrac{C_p\mathrm{Le}}{r_a d}\left(\dfrac{1}{\sqrt{\pi\,\mathrm{Fo}}} - 1\right)}; \tag{10}$$

$$\tilde{C}_s = \frac{dT_0}{C_0}\,\tilde{T}_s + \frac{b}{C_0}. \tag{11}$$

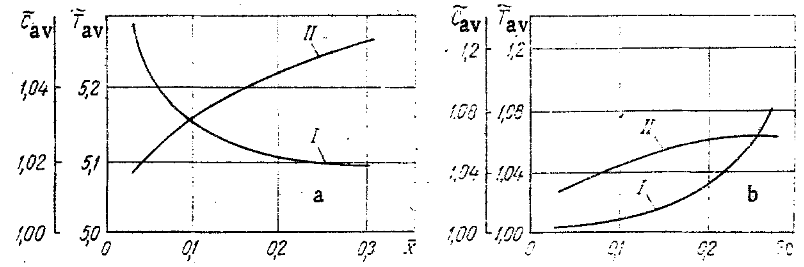

Fig. 2. Dependence of mean bulk temperature and concentration in a drop on Fo and of mean cross-sectional temperature and concentration in a film on $\bar{x} = x/\mathrm{RePr}\delta$; a) film; b) drop; I) $\tilde{T}_{av}$; II) $\tilde{C}_{av}$; $dT_0/C_0 = -0.5$; Le = 25; $b/C_0 = 1.7$; Ka = −0.1; $T_w/T_0 = 0.5$.

Using Eqs. (8)-(11), expressions can be obtained for the dimensionless flows of heat and mass at the surface of the drop:

$$Q_{\text{т}} = \frac{g_{\text{т}} R}{\lambda T_0} = \frac{R}{T_0} \left.\frac{\partial T}{\partial r}\right|_{r=R} = (\tilde{T}_s - 1)\left(\frac{1}{\sqrt{\pi \mathrm{Fo}}} - 1\right); \tag{12}$$

$$Q_{\text{м}} = \frac{g_{\text{м}} R}{\rho D C_0} = (\tilde{C}_s - 1)\left(\frac{\sqrt{\mathrm{Le}}}{\sqrt{\pi \mathrm{Fo}}} - 1\right), \tag{13}$$

and also for the mean bulk temperature and concentration:

$$\tilde{T}_{av} = \frac{T_{av}}{T_0} = \frac{4\pi}{V}\int_0^R r^2 \tilde{T}(r,\, t)\, dr = 3(\tilde{T}_s - 1)\left[\left(-\frac{1}{2} - \mathrm{Fo}\right) \times \right.$$

$$\left. \times \operatorname{erf}\frac{1}{2\sqrt{\mathrm{Fo}}} - \frac{\sqrt{\mathrm{Fo}}}{\sqrt{\pi}}\left(\exp\left(-\frac{1}{4\mathrm{Fo}}\right) - 1\right) + \frac{\sqrt{\mathrm{Fo}}}{\sqrt{\pi}}\right] + \frac{3}{2}(\tilde{T}_s - 1) + 1; \tag{14}$$

$$\tilde{C}_{av} = \frac{C_{av}}{C_0} = \frac{4\pi}{V}\int_0^R r^2 \tilde{C}(t,\, r)\, dr = 3(\tilde{C}_s - 1)\left\{\left(-\frac{1}{2} - \frac{\mathrm{Fo}}{\mathrm{Le}}\right) \times \right.$$

$$\left. \times \operatorname{erf}\frac{\sqrt{\mathrm{Le}}}{2\sqrt{\mathrm{Fo}}} - \frac{\sqrt{\mathrm{Fo}}}{\sqrt{\pi}\sqrt{\mathrm{Le}}}\left[\exp\left(-\frac{\mathrm{Le}}{4\mathrm{Fo}}\right) - 1\right] + \frac{\sqrt{\mathrm{Fo}}}{\sqrt{\pi \mathrm{Le}}}\right\} + \frac{3}{2}(\tilde{C}_s - 1) + 1. \tag{15}$$

As is clear from Eqs. (7)-(15), heat and mass transfer inside a drop are determined by the numbers Le and Fo, by Ka = dr_a/C_p, and by the concentration head ($dT_0 + b - C_0$). The quantity Ka is an analog of the phase transition criterion, since 1/d has the dimensionality of temperature. Such a quantity was used by Sparrow and Spalding [1] in the problem of sublimation in a channel.

We investigate the relation for the dimensionless flow of mass, which is one of the basic characteristics of the process.

Transforming Eq. (13) by means of Eqs. (10) and (11), we obtain

$$\tilde{Q}_{\text{м}} = \frac{R g_{\text{м}}}{\rho D (dT_0 + b - C_0)} = \frac{\mathrm{Le}}{\sqrt{\pi \mathrm{Fo}}\left(\dfrac{1}{\sqrt{\mathrm{Le}} - \sqrt{\pi \mathrm{Fo}}} - \dfrac{\mathrm{Ka}}{1 - \sqrt{\pi \mathrm{Fo}}}\right)}. \tag{16}$$

The form of Eq. (16) indicates that this expression is only valid for Fo < $1/\pi$. When Fo = $1/\pi$, $\tilde{Q}_M = 0$, i.e., the absorption process ceases.

We now consider the problem of combined heat and mass transfer during absorption of vapor by a liquid film flowing over an impermeable isothermal well.

The fluid flow velocity v in the film and the film thickness δ are assumed constant. In a Cartesian coordinate system (x, y) with the x axis having the same direction as the velocity v and lying along the wall, and with the origin on the wall at the initial cross section of the film, the process is described by the system of equations

$$v\frac{\partial T}{\partial x} = a\frac{\partial^2 T}{\partial y^2},$$

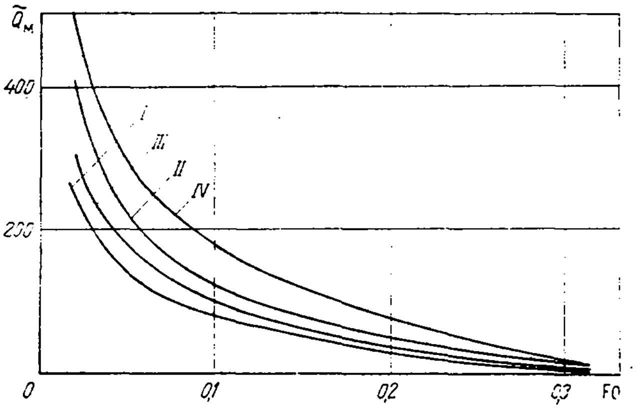

Fig. 3. Dependence of mass flow at drop surface on Fo for various values of Ka and Le ($dT_0/C_0 = -0.5$; $b/C_0 = 1.7$): I) Ka = −1/7, Le = 25; II) Ka = −1/7, Le = 37; III) Ka = −1/12, Le = 25; IV) Ka = −1/12, Le = 37.

$$v\,\frac{\partial C}{\partial x} = D\,\frac{\partial^2 C}{\partial y^2} \tag{17}$$

with the boundary conditions

$$T(0,\ y) = T_0; \tag{18}$$

$$C(0,\ y) = C_0; \tag{19}$$

$$T(x,\ 0) = T_W; \tag{20}$$

$$\left.\frac{\partial C}{\partial y}\right|_{y=0} = 0; \tag{21}$$

$$C(x,\ \delta) = dT(x,\ \delta) + b; \tag{22}$$

$$\left.\frac{\partial C}{\partial y}\right|_{y=\delta} = \frac{C_p\,\mathrm{Le}}{d}\,\left.\frac{\partial T}{\partial y}\right|_{y=\delta}. \tag{23}$$

The necessary physical assumptions used in Eqs. (17)-(23) were adapted from the preceding problem.

Evaluations of the exact solution of Eqs. (17)-(23) show that the temperature profile is nearly linear in each transverse section of the film. Therefore one can reasonably assume

$$\tilde{T} = \frac{T}{T_0} = (\tilde{T}_s - \tilde{T}_W)\,\bar{y} + \tilde{T}_W. \tag{24}$$

Then Eq. (24) formally satisfies Eq. (17).

Now one can consider the approximation of an asymptotic surface layer for concentration as was done in the preceding problem. The corresponding self-similar solution has the form

$$\tilde{C} = \frac{C}{C_0} = (1 - \tilde{C}_s)\,\mathrm{erf}\,\eta + \tilde{C}_s. \tag{25}$$

Using saturation condition (22) and Eq. (23) for the heat flow at the surface of the film for $\tilde{T}_s$ and $\tilde{C}_s$, the following expressions can be found:

$$\tilde{T}_s = \frac{T_s}{T_0} = \frac{\sqrt{\mathrm{Le}}\,\tilde{T}_W - \mathrm{Ka}\,\dfrac{C_0}{dT_0}\left(\dfrac{b}{C_0} - 1\right)\dfrac{1}{\sqrt{\pi x}}}{\sqrt{\mathrm{Le}} - \mathrm{Ka}\,\dfrac{1}{\sqrt{\pi x}}}; \tag{26}$$

$$\tilde{C}_s = \frac{dT_0}{C_0}\,\tilde{T}_s + \frac{b}{C_0}. \tag{27}$$

Expressions for the average values over a cross section follow from Eqs. (24)-(27):

$$\tilde{T}_{\mathrm{av}} = \frac{T_{\mathrm{av}}}{T_0} = \frac{1}{\delta T_0}\int_0^{\delta} T\,dy = \frac{\tilde{T}_s + \tilde{T}_W}{2}; \tag{28}$$

$$\tilde{C}_{\mathrm{av}} = \frac{C_{\mathrm{av}}}{C_0} = \frac{1}{\delta C_0}\int_0^{\delta} C\,dy = (1-\tilde{C}_s)\left[\operatorname{erf}\frac{\sqrt{\mathrm{Le}}}{2\sqrt{\bar{x}}} + \frac{2\sqrt{\bar{x}}}{\sqrt{\pi\,\mathrm{Le}}}\left(\exp\left(-\frac{\mathrm{Le}}{4\bar{x}}\right)-1\right)\right] + \tilde{C}_s. \tag{29}$$

For the dimensionless flows of heat and mass we have, respectively,

$$Q_T = \tilde{T}_s - \tilde{T}_W; \tag{30}$$

$$Q_M = (\tilde{C}_s - 1)\frac{\sqrt{\mathrm{Le}}}{\sqrt{\pi}\sqrt{\bar{x}}}. \tag{31}$$

Transforming Eq. (31) by means of Eqs. (26) and (27) leads to

$$Q_M = \frac{\delta g_M}{\rho D(dT_W + b - C_0)} = \frac{\mathrm{Le}}{\pi^{1/2}\,\mathrm{Le}^{1/2}\mathrm{Re}^{-1/2}\,\mathrm{Pr}^{-1/2}\left(\frac{x}{\delta}\right)^{1/2} - \mathrm{Ka}}. \tag{32}$$

It is clear from Eqs. (16) and (32) that in the first case the flow of absorbable material normalized to the concentration head is determined by a system of the three numbers Fo, Le, and Ka, and in the second case by a system of the four numbers Re, Pr, Le, and Ka. Equations (8)-(16) and (24)-(32) were used for calculations in the parameter range typical of bromine—lithium absorbers.

The calculated curves are shown in Figs. 1-3. The results can be particularly useful in the analysis of experimental data for the absorption of vapor in liquid drops and films.

NOTATION

r, distance measured from center of drop; a, coefficient of thermal diffusivity; D, diffusion coefficient; λ, coefficient of thermal conductivity; r_a, heat of absorption including heat of phase transition and heat of solution; d, b, constants depending on saturation pressure; R, radius of drop; ρ, density of fluid; η_1, η_2, η, self-similar variables [$\eta_1 = (r - R)/2\sqrt{at}$; $\eta_2 = (r - R)/2\sqrt{Dt}$; $\eta = \sqrt{\mathrm{Le}}(1 - \bar{y})/2\sqrt{\bar{x}}$]; Le $= a/D$, Lewis number; Fo, Fourier number; Re $= v\delta/\nu$, Reynolds number; Pr $= \nu/a$, Prandtl number; g_T, g_M dimensional flows of heat and mass; V, volume of drop; $\bar{y} = y/\delta$; $\bar{x} = xa/v\delta^2$.

LITERATURE CITED

1. E. M. Sparrow and E. C. Spalding, J. Heat Transfer, 90, No. 1 (1968).